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ORIGINAL RESEARCH

Fixed point results for α-ψ-locally graphic contraction in dislocated qusai metric spaces

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Abstract In this paper, we have obtained fixed point results for $\alpha-\psi$ -locally contractive type mappings in a closed ball in left K-sequentially complete and in right K-sequentially complete dislocated quasi metric spaces. Moreover the mappings under consideration are α-admissible with respect to η. We have used conditions weaker than those of Samet et al. [Nonlinear Anal. 75:2154–2165, (2012)]. As an application, we have derived some new fixed point theorems for ψ -graphic contractions defined on dislocated quasi metric space endowed with a graph as well as ordered dislocated metric space. Some comparative examples are constructed which illustrate the superiority of our results. In the process we have generalized several well known, recent and classical results from the literature.

Keywords Fixed point · Complete dislocated quasi metric space · Left K -sequentially complete · α-ψ contractive mappings · ψ -graphic contractions · Closed ball.

Introduction and preliminaries

Fixed point theory has wide and endless applications in many fields of engineering and science. Its core, the Banach contraction principle, has attracted many researchers who tried to generalize it in different aspects. Fixed point results of mappings satisfying certain contractive condition on the entire domain has been at the centre of rigorous research activities.

From the application point of view the situation is not yet completely satisfactory because it frequently happens that a mapping T is a contraction not on the entire space X but merely on a subset Y of X. However, we impose subtle restrictions to obtain fixed point results for such mapping. Recently Arshad et al. [7] proved a result concerning the existence of fixed points of a mapping satisfying a contractive conditions on closed ball in a complete dislocated metric space. Other results on closed ball can be seen in [6, 8–11, 27, 35, 36]. Recently, Karapınar et al. [24] introduced the concept of quasi-partial metric space. Zeyada et al. [37] introduced the concept of dislocated quasi metric which is basically the generalization of quasi-partial metric space.

Recently, Samet et al. [34] introduced the notions of α-ψ -contractive and α-admissible mapping in complete metric spaces. The existance of fixed points of α-ψ-contractive and α -admissible mapping in complete metric spaces has been studied by several researchers, (see [3, 4, 19, 20, 25, 26, 33]).

Consistent with [33, 34, 37], we give the following definitions which will be needed in the sequel.

Definition 1.1 [37] Let X be a nonempty set. Let $d_q : X \times X \to [0,\infty)$ be a function, called a dislocated qusai metric (or simply d_q-metric) if the following conditions hold for any $x, y, z \in X$:

(i) If $d_q(x, y) = d_q(y, x) = 0$, then $x = y$,
(ii) $d_q(x, y) \leq d_q(x, z) + d_q(z, y)$.

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The pair (X, d_q) is called a dislocated qusai metric space. It is clear that if $d_q(x, y) = d_q(y, x) = 0$, then from (i), $x = y$. But if $x = y$, $d_q(x, y)$ may not be 0. It is observed that if $d_q(x, y) = d_q(y, x)$ for all $x, y \in X$, then (X, d_q) becomes a dislocated metric space. We shall denote (X, d_l) for a dislocated metric space. The ball $\overline{B(x, \varepsilon)}$ where $\overline{B(x, \varepsilon)} = \{y \in X : d_q(x, y) \leq \varepsilon\}$ is a closed ball in dislocated qusai metric space, for some $x \in X$ and $\varepsilon > 0$. It is clear that any qusai-partial metric is a d_q-metric.

Example 1.2 If $X = R^+ \cup \{0\}$ then $d_q(x, y) = x + 2y$ defines a dislocated quasi metric d_q on X.

Example 1.3 If $X = R^+ \cup \{0\}$ then $d_q(x, y) = x + \max\{x, y\}$ defines a dislocated quasi metric d_q on X.

Let Ψ denote the family of all nondecreasing functions $\psi : [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=1}^{+\infty} \psi^n(t) < +\infty$ for all $t > 0$, where ψ^n is the n^{th} itrerate of ψ.

Lemma 1.4 [33] *If $\psi \in \Psi$, then $\psi(t) < t$ for all $t > 0$.*

Definition 1.5 [34] Let (X, d) be a metric space and $S : X \to X$ be a given mapping. We say that S is α-ψ contractive mapping if there exist two functions $\alpha : X \times X \to [0, +\infty)$ and $\psi \in \Psi$ such that $\alpha(x, y)d(Sx, Sy) \leq \psi(d(x, y))$ for all $x, y \in X$.

Remark 1.6 [33] By definition, $\alpha(x, x) \neq 0$ for $x \in X$.

Definition 1.7 [33] Let $S : X \to X$ and $\alpha, \eta : X \times X \to [0, +\infty)$ be two functions. We say that S is α-admissible mapping with respect to η if $x, y \in X$ such that $\alpha(x, y) \geq \eta(x, y)$ then we have $\alpha(Sx, Sy) \geq \eta(Sx, Sy)$. Note that if we take $\eta(x, y) = 1$, then this definition reduces to Definition 1.1 of [33]. Also, if we take $\alpha(x, y) = 1$, then we say that T is an η-subadmissible mapping.

In this paper, we shall prove a theorem which is an extension of the results of Samet et al. [34].

Main results

Reilly et al. [31] introduced the notion of left (right) K-Cauchy sequence and left (right) K-sequentialy complete spaces(see [11, 16]). We use this concept to establish the following definition.

Definition 2.1 Let (X, d_q) be a dislocated quasi metric space.

(a) A sequence $\{x_n\}$ in (X, d_q) is called left (right) K-Cauchy if $\forall\ \varepsilon > 0$, $\exists\ n_0 \in N$ such that $\forall\ n > m \geq n_0$, $d_q(x_m, x_n) < \varepsilon$ (respectively $d_q(x_n, x_m) < \varepsilon$).

(b) A sequence $\{x_n\}$ dislocated quasi-converges (for short d_q -converges) to x if $\lim_{n\to\infty} d_q(x_n, x) = \lim_{n\to\infty} d_q(x, x_n) = 0$. In this case x is called a d_q-limit of $\{x_n\}$.

(c) (X, d_q) is called left (right) K-sequentially complete if every left (right) K-Cauchy sequence in X converges to a point $x \in X$ such that $d_q(x, x) = 0$.

One can easily observe that every complete dislocated quasi metric space is also left K-sequentially complete dislocated quasi metric space but the converse is not true in general.

Theorem 1 *Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space. Suppose there exist two functions, $\alpha, \eta : X \times X \to [0, +\infty)$. Let x_0 be an arbitrary point in X and $S : X \to X$ be α-admissible with respect to η and $\psi \in \Psi$. Assume that,*

$$x, y \in \overline{B(x_0, r)}, \alpha(x, y) \geq \eta(x, y) \Longrightarrow d_q(Sx, Sy) \leq \psi(d_q(x, y)) \tag{1}$$

and

$$\sum_{i=0}^{j} \psi^i(d_q(x_0, Sx_0)) \leq r, \text{ for all } j \in N \text{ and } r > 0. \tag{2}$$

Suppose that the following assertions hold:

(i) $\alpha(x_0, Sx_0) \geq \eta(x_0, Sx_0)$;

(ii) *for any sequence $\{x_n\}$ in $\overline{B(x_0, r)}$ such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ for all $n \in N \cup \{0\}$ and $x_n \to u \in \overline{B(x_0, r)}$ as $n \to +\infty$ then $\alpha(x_n, u) \geq \eta(x_n, u)$ for all $n \in N \cup \{0\}$.*

Then, there exists a point x^* in $\overline{B(x_0, r)}$ such that $x^* = Sx^*$.

Proof Choose a point x_1 in X such that $x_1 = Sx_0$ let $x_2 = Sx_1$. Continuing this process, we construct a sequence x_n of points in X such that,

$$x_{i+1} = Sx_i, \quad \text{where i} = 0, 1, 2, \ldots$$

□

By assumption $\alpha(x_0, x_1) \geq \eta(x_0, x_1)$ and S is α-admissible with respect to η, we have, $\alpha(Sx_0, Sx_1) \geq \eta(Sx_0, Sx_1)$ we deduce that $\alpha(x_1, x_2) \geq \eta(x_1, x_2)$ which also implies that $\alpha(Sx_1, Sx_2) \geq \eta(Sx_1, Sx_2)$. Continuing in this way obtain $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ for all $n \in N \cup \{0\}$. First we show that $x_n \in \overline{B(x_0, r)}$ for all $n \in N$. Using inequality (2), we have,

$$\sum_{i=0}^{n} \psi^i(d_q(x_0, Sx_0)) \leq r \quad \text{for all}\, j \in N.$$

It follows that,

$$x_1 \in \overline{B(x_0, r)}.$$

Let $x_2, \ldots, x_j \in \overline{B(x_0, r)}$ for some $j \in N$. so using inequality (1), we obtain,

$$\begin{aligned} d_q(x_j, x_{j+1}) &= d_q(Sx_{j-1}, Sx_j) \\ &\le \psi(d_q(x_{i-1}, x_i)) \\ &\le \psi^2(d_q(x_{j-2}, x_{j-1})) \\ &\le \cdots \le \psi^j(d_q(x_0, x_1)). \end{aligned}$$

Thus we have,

$$d_q(x_j, x_{j+1}) \le \psi^j(d_q(x_0, x_1)). \tag{3}$$

Now,

$$\begin{aligned} d_q(x_0, x_{j+1}) &= d_q(x_0, x_1) + d_q(x_1, x_2) + d_q(x_2, x_3) + \ldots \\ &\quad + d_q(x_j, x_{j+1}) \\ &\le \sum_{i=0}^{j} \psi^i(d_q(x_0, x_1)) \\ &\le r. \end{aligned}$$

Thus $x_{j+1} \in \overline{B(x_0, r)}$. Hence $x_n \in \overline{B(x_0, r)}$ for all $n \in N$. Now inequality (3) can be written as

$$d_q(x_n, x_{n+1}) \le \psi^n(d_q(x_0, x_1)), \quad \text{for all n} \in \mathrm{N}. \tag{4}$$

Fix $\varepsilon > 0$ and let $n(\varepsilon) \in N$ such that $\sum \psi^n(d_q(x_0, x_1)) < \varepsilon$. let $n, m \in N$ with $m > n > n(\varepsilon)$ using the triange inequality, we obtain,

$$\begin{aligned} d_q(x_n, x_m) &\le \sum_{k=n}^{m-1} d_q(x_k, x_{k+1}) \le \sum_{k=n}^{m-1} \psi^k(d_q(x_0, x_1)) \\ &\le \sum_{n \ge n(\varepsilon)} \psi^k(d_q(x_0, x_1)) < \varepsilon. \end{aligned}$$

Thus we have proved that $\{x_n\}$ is a left K-Cauchy sequence in $(\overline{B(x_0, r)}, d_q)$. As $\overline{B(x_0, r)}$ is closed, so it is left K-sequentially complete. Therefore, there exists a point $x^* \in \overline{B(x_0, r)}$ such that $x_n \to x^*$. Also

$$\lim_{n\to\infty} d_q(x_n, x^*) = 0. \tag{5}$$

On the other hand, from (ii), we have,

$$\alpha(x^*, x_n) \ge \eta(x^*, x_n) \quad \text{for all } n \in N \cup \{0\}. \tag{6}$$

Now using the triangle inequality, also by using (1) and (6), we get

$$d_q(Sx^*, x_{i+1}) \le \psi(d_q(x^*, x_i)) < d_q(x^*, x_i). \tag{7}$$

Letting $i \to \infty$ and by using inequality (7), we obtain $d_q(Sx^*, x^*) < 0$. Hence $Sx^* = x^*$.

If $\eta(x, y) = 1$ for all $x, \ y \in X$ in Theorem 2.2, we obtain following result.

Corollary 2 *Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space. Suppose there exist a function, $\alpha : X \times X \to [0, +\infty)$. Let x_0 be an arbitrary point in X and $S : X \to X$ be α-admissible and $\psi \in \Psi$. Assume that,*

$$x, y \in \overline{B(x_0, r)}, \alpha(x, y) \ge 1 \Longrightarrow d_q(Sx, Sy) \le \psi(d_q(x, y))$$

and

$$\sum_{i=0}^{j} \psi^i(d_q(x_0, Sx_0)) \le r, \quad \text{forall } j \in N \text{ and } r > 0.$$

Suppose that the following assertions hold:

(i) $\alpha(x_0, Sx_0) \ge 1$;
(ii) *for any sequence $\{x_n\}$ in $\overline{B(x_0, r)}$ such that $\alpha(x_n, x_{n+1}) \ge 1$ for all $n \in N \cup \{0\}$ and $x_n \to u \in \overline{B(x_0, r)}$ as $n \to +\infty$ then $\alpha(x_n, u) \ge 1$ for all $n \in N \cup \{0\}$.*

Then, there exists a point x^* in $\overline{B(x_0, r)}$ such that $x^* = Sx^*$.

Theorem 3 *Adding condition "if x and y are any fixed point in $\overline{B(x_0, r)}$ then $\alpha(x, y) \ge \eta(x, y)$" to the hypotheses of Theorem 2.2. Then S has a unique fixed point x^* and $d_q(x^*, x^*) = 0$.*

Proof Assume that x^* and y^* be two fixed point of S in $\overline{B(x_0, r)}$, then, by assumption, $\alpha(x^*, y^*) \ge \eta(x^*, y^*)$,

$$d_q(x^*, y^*) = d_q(Sx^*, Sy^*) \le \psi(d_q(x^*, y^*))$$

A contradiction to the fact that for each $t > 0$, $\psi(t) < t$. So $x^* = y^*$. Hence S has no fixed point other than x^*. Now, $\alpha(x^*, x^*) \ge \eta(x^*, x^*)$, then,

$$d_q(x^*, x^*) = d_q(Sx^*, Sx^*) \le \psi(d_q(x^*, x^*)).$$

This implies that,

$$d_q(x^*, x^*) = 0.$$

□

Example 2.2 Let $X = Q^+ \cup \{0\}$ and let $d_q : X \times X \to X$ be the complete ordered dislocated quasi metric on X defined by $d_q(x, y) = x + 2y$, endowed with usual order. Let $S : X \to X$ be defined by,

$$Sx = \begin{cases} \dfrac{x}{7} & \text{if} x \in [0, 1] \\ x - \dfrac{1}{2} & \text{if} x \in (1, \infty) \end{cases}$$

$x_0 = 1$, $r = 3$, $\overline{B(x_0, r)} = [0, 1]$ and $\alpha(x, y) = 3$ for all x, y. Clearly S is an α-ψ-contractive mapping, where $\psi(t) = \frac{t}{3}$.

$$d_q(1, S1) = d_q\left(1, \frac{1}{7}\right) = 1 + \frac{2}{7} = \frac{9}{7}$$

$$\sum_{i=1}^{n} \psi^n(d_q(x_0, Sx_0)) = \frac{9}{7}\sum_{i=1}^{n} \frac{1}{3^n} < \frac{3}{2}\left(\frac{9}{7}\right) = \frac{27}{14} < 3$$

If $x, y \in \overline{B(x_0, r)}$, then

$$\frac{3x}{7}+\frac{6y}{7}\le x+2y$$
$$\frac{x}{7}+\frac{2y}{7}\le\frac{x+2y}{3}$$
$$d_q(Sx,\ Sy)\le\psi(d_q(x,\ y))$$

Also if $x,\ y\in(1,\ \infty)$, then

$$3x+6y-\frac{9}{2}>x+2y$$
$$x+2y-\frac{3}{2}>\frac{x+2y}{3}$$
$$\left(x-\frac{1}{2}\right)+2\left(y-\frac{1}{2}\right)>\psi(x+2y)$$
$$d_q(Sx,\ Sy)>\psi(d_q(x,\ y))$$

Then the contractive condition does not hold on X.

Therefore, all the conditions of corollary 2.3 are satisfied and 0 is the unique fixed point of S.

Corollary 4 *Let $(X,\ p_q)$ be a left K-sequentially complete partial quasi metric space. Suppose there exist two functions, $\alpha,\eta : X\times X\to[0,\ +\infty)$. Let x_0 be an arbitrary point in X and $S : X\to X$ be α-admissible with respect to η and $\psi\in\Psi$. Assume that,*

$$x,\ y\in\overline{B(x_0,\ r)},\alpha(x,\ y)\ge\eta(x,\ y)\Longrightarrow p_q(Sx,\ Sy)\le\psi(p_q(x,\ y))$$

and

$$\sum_{i=0}^{j}\psi^i(p_q(x_0,Sx_0))\le r+p(x_0,\ x_0)\quad\text{for all } j\in N \text{ and } r>0.$$

Suppose that, the following assertions hold:

(i) $\alpha(x_0,\ Sx_0)\ge\eta(x_0,\ Sx_0)$;

(ii) for any sequence $\{x_n\}$ in $\overline{B(x_0,\ r)}$ such that $\alpha(x_n,\ x_{n+1})\ge\eta(x_n,\ x_{n+1})$ for all $n\in N\cup\{0\}$ and $x_n\to u\in\overline{B(x_0,\ r)}$ as $n\to+\infty$ then $\alpha(x_n,\ u)\ge\eta(x_n,\ u)$ for all $n\in N\cup\{0\}$.

Then, there exists a point x^* in $\overline{B(x_0,\ r)}$ such that $x^*=Sx^*$.

Fixed point results for graphic contractions in dislocated quasi metric spaces

Consistent with Jachymski [23], let $(X,\ d_q)$ be a dislocated quasi metric space and Δ denotes the diagonal of the Cartesian product $X\times X$. Consider a directed graph G such that the set $V(G)$ of its vertices coincides with X, and the set $E(G)$ of its edges contains all loops, i.e., $E(G)\supseteq\Delta$. We assume G has no parallel edges, so we can identify G with the pair $(V(G),\ E(G))$. Moreover, we may treat G as a weighted graph (see [23]) by assigning to each edge the distance between its vertices. If x and y are vertices in a graph G, then a path in G from x to y of length m $(m\in\mathbb{N})$ is a sequence $\{x_i\}_{i=0}^{m}$ of $m+1$ vertices such that $x_0=x$, $x_m=y$ and $(x_{n-1},\ x_n)\in E(G)$ for $i=1,\ldots,m$. A graph G is connected if there is a path between any two vertices. G is weakly connected if $\tilde{G}$ is connected (see for details [1, 13, 21, 23]).

Definition 3.1 [23] We say that a mapping $T : X\to X$ is a Banach G-contraction or simply G-contraction if T preserves edges of G, i.e.,

$$\forall x\ ,y\in X((x,\ y)\in E(G)\Rightarrow(Tx,\ Ty)\in E(G))$$

and T decreases weights of edges of G in the following way:

$$\exists k\in(0,\ 1),\forall x\ ,y\in X((x,y)\in E(G)\Rightarrow d(Tx,Ty)\le kd(x,y)).$$

Definition 3.2 Let $(X,\ d_q)$ be a dislocated quasi metric space endowed with a graph G and $S : X\to X$ be self-mapping. Assume that for $r>0$, $x_0\in X$ and $\psi\in\Psi$, following conditions hold,

$$\forall x,\ y\in\overline{B(x_0,\ r)}\ ((x,\ y)\in E(G)\Rightarrow(Sx,\ Sy)\in E(G).$$

$$\forall x,\ y\in\overline{B(x_0,\ r)},\ (\mathrm{x},\ \mathrm{y})\in \mathrm{E(G)}\Rightarrow \mathrm{d_q}(\mathrm{Sx},\ \mathrm{Sy})\le\psi(\mathrm{d_q}(\mathrm{x},\ \mathrm{y})).$$

Then the mapping S is called a ψ-graphic contractive mapping. If $\psi(t)=kt$ for some $k\in[0,\ 1)$, then we say S is G -contractive mappings.

Theorem 5 *Let $(X,\ d_q)$ be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and $S : X\to X$ be ψ-graphic contractive mapping. Suppose that the following assertions hold:*

Definition 3.3

(i) $(x_0,\ Sx_0)\in E(G)$ and $\sum_{i=0}^{j}\psi^i(d_q(x_0,\ Sx_0))\le r$ for all $j\in N$ and $r>0$.

(ii) if $\{x_n\}$ is a sequence in $\overline{B(x_0,\ r)}$ such that $(x_n,\ x_{n+1})\in E(G)$ for all $n\in\mathbb{N}$ and $x_n\to x$ as $n\to+\infty$, then $(x_n,\ x)\in E(G)$ for all $n\in\mathbb{N}$.

Then S has a fixed point.

Proof Define, $\alpha : X^2\to(-\infty,\ +\infty)$ by $\alpha(x,\ y)=\begin{cases}1, & \text{if }\ (x,\ y)\in E(G)\\ 0, & \text{otherwise}\end{cases}$. At fist we prove that the mapping S is α-admissible. Let $x,\ y\in\overline{B(x_0,\ r)}$ with $\alpha(x,\ y)\ge 1$, then $(x,\ y)\in E(G)$. As S is ψ-graphic contractive mappings, we have, $(Sx,\ Sy)\in E(G)$. That is, $\alpha(Sx,\ Sy)\ge 1$. Thus S is α-admissible mapping. From (i) there exists x_0 such that $(x_0,\ Sx_0)\in E(G)$. That is, $\alpha(x_0,\ Sx_0)\ge 1$. If $x,\ y\in\overline{B(x_0,\ r)}$ with $\alpha(x,\ y)\ge 1$, then $(x,\ y)\in E(G)$. Now, since S, is ψ -graphic contractive mapping, so $d_q(Sx, Sy) \leq \psi(d_q(x, y))$. That is,

$$\alpha(x, y) \geq 1 \Longrightarrow d_q(Sx, Sy) \leq \psi(d_q(x, y)).$$

Let $\{x_n\} \subset \overline{B(x_0, r)}$ with $x_n \to x$ as $n \to \infty$ and $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. Then, $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$ and $x_n \to x$ as $n \to +\infty$. So by (ii) we have, $(x_n, x) \in E(G)$ for all $n \in \mathbb{N}$. That is, $\alpha(x_n, x) \geq 1$. Hence, all conditions of Theorem 1 are satisfied and S has a fixed point. □

Theorem 3.2 (2^o) [23] and corollary 2.3(2)[14] are extended to ψ-graphic contractive defined on a dislocated quasi metric space as follows.

Corollary 6 *Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and $S : X \to X$ be ψ-graphic contractive mapping. Suppose that the following assertions hold:*

(i) $(x_0, Sx_0) \in E(G)$ and $\sum_{i=0}^{j} \psi^i(d_q(x_0, Sx_0)) \leq r$ for all $j \in N$ and $r > 0$.

(ii) $(x, z) \in E(G)$ and $(z, y) \in E(G)$ imply $(x, y) \in E(G)$ for all $x, y, z \in X$, that is, $E(G)$ is a quasi-order [23] and if $\{x_n\}$ is a sequence in $\overline{B(x_0, r)}$ such that $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$ and $x_n \to x$ as $n \to +\infty$, then there is a subsequence $\{x_{k_n}\}$ with $(x_{k_n}, x) \in E(G)$ for all $n \in \mathbb{N}$.

Then S has a fixed point.

Proof Condition (ii) implies that of (*ii*) in Theorem 5 (see Remark 3.1 [23]). Now the conclusion follows from Theorem 5. □

Corollary 7 *Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and $S : X \to X$ be a mapping. Suppose that the following assertions hold:*

(i) S is Banach G-contraction on $\overline{B(x_0, r)}$;

(ii) $(x_0, Sx_0) \in E(G)$ and $d_q(x_0, Sx_0)) \leq (1 - k)r$;

(iii) if $\{x_n\}$ is a sequence in $\overline{B(x_0, r)}$ such that $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$ and $x_n \to x$ as $n \to +\infty$, then $(x_n, x) \in E(G)$ for all $n \in \mathbb{N}$.

Then S has a fixed point.

Corollary 8 *Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and $S : X \to X$ be a mapping. Suppose that the following assertions hold:*

(i) S is Banach G-contraction on X and there is $x_0 \in X$ such that $(x_0, Sx_0) \in E(G)$;

(iii) if $\{x_n\}$ is a sequence in X such that $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$ and $x_n \to x$ as $n \to +\infty$, then $(x_n, x) \in E(G)$ for all $n \in \mathbb{N}$.

Then S has a fixed point.

The study of existence of fixed points in partially ordered sets has been initiated by Ran and Reurings [30] with applications to matrix equations. Agarwal, et al. [2], Bhaskar and Lakshmikantham [12], Ciric et al. [15] and Hussain et al. [22] presented some new results for nonlinear contractions in partially ordered metric spaces and noted that their theorems can be used to investigate a large class of problems. Here as an application of our results we deduce some new common fixed point results in partially ordered dislocated quasi metric spaces.

Recall that if $(X, \preceq)$ is a partially ordered set and $S : X \to X$ is such that for $x, y \in X$, with $x \preceq y$ implies $Sx \preceq Sy$, then the mapping S is said to be non-decreasing.

Let $(X, d_q, \preceq)$ be a partially ordered dislocated quasi metric space. Define the graph G by

$$E(G) := \{(x, y) \in X \times X : x \preceq y\}.$$

For this graph, first condition in Definition 3.2 means S is non-decreasing with respect to this order. We derive following important results in partially ordered dislocated quasi metric spaces.

Corollary 9 *Let $(X, \preceq, d_q)$ be a partially ordered left K-sequentially complete dislocated quasi metric space and $S : X \to X$ be a nondecreasing map. Suppose that the following assertions hold:*

(i) there exists $k \in [0, 1)$ such that $d_q(Sx, Sy) \leq kd_q(x, y)$ for all $x, y \in \overline{B(x_0, r)}$ with $x \preceq y$;

(ii) $x_0 \preceq Sx_0$ and $d_q(x_0, Sx_0) \leq (1 - k)r$;

(iii) if $\{x_n\}$ is a nondecreasing sequence in $\overline{B(x_0, r)}$ such that $x_n \to x \in \overline{B(x_0, r)}$ as $n \to +\infty$, then $x_n \preceq x$ for all n.

Then S has a fixed point.

Corollary 10 *Let $(X, \preceq, d_q)$ be a partially ordered left K-sequentially complete dislocated quasi metric space and $S : X \to X$ be a nondecreasing map. Suppose that the following assertions hold:*

(i) there exists $k \in [0, 1)$ such that $d_q(Sx, Sy) \leq kd_q(x, y)$ for all $x, y \in X$ with $x \preceq y$;

(ii) there exists $x_0 \in X$ such that $x_0 \preceq Sx_0$;

(iii) if $\{x_n\}$ is a nondecreasing sequence in X such that $x_n \to x \in X$ as $n \to +\infty$, then $x_n \preceq x$ for all n.

Then S has a fixed point.

Corollary 11 ([29]) *Let $(X, \preceq, d)$ be a partially ordered complete metric space and $S : X \to X$ be a nondecreasing mapping such that*

$d(Sx, Sy) \le kd(x, y)$

for all $x, y \in X$ with $x \preceq y$ where $0 \le k < 1$. Suppose that the following assertions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Sx_0$;
(ii) if $\{x_n\}$ is a sequence in X such that $x_n \preceq x_{n+1}$ for all $n \in \mathbb{N}$ and $x_n \to x$ as $n \to +\infty$, then $x_n \preceq x$ for all $n \in \mathbb{N}$.

Then S has a fixed point.

Remark The above results can easily be proved in right K-sequentially dislocated quasi metric space.

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